Trovare un p. scalare $*: \mathbb{R}^2 \to \mathbb{R}^2$ $(-1,1)$ $(1,0)$ ortonorm. rispetto $*$

$$* \quad \begin{bmatrix} a & b \\ b & c \end{bmatrix} = A$$

$$[-1, 1] A \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 1 \qquad [-a+b \quad -b+c] \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \cancel{0}\, 1$$

$$[1 \ 0] A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \qquad [a \ b] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \to a = 1$$

$$[-1 \ 1] A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 \qquad -a+b = 0 \Rightarrow b = a = 1$$

$$\cancel{a-b-b+c=0} \quad \cancel{b-a-b+c} = \cancel{c} = a$$

$$\cancel{-1+c=0} \to c = 1$$

$$\begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

$$b-a-b+c=1$$

$$\begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}a & b\\ b & a\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix}=1$$

$$c = 1+a$$

$$c=2$$

$$[1\,0] * [1,0] = 1$$

$$\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix}=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix}=1 \quad \text{ok}$$

$$\begin{bmatrix}-1 & 1\end{bmatrix}A\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}=0 \quad \underline{\underline{\text{ok}}}$$

$\pi: ix+y-2z-3i=0$ $\quad r$ luogo punti reali

π piano in $\mathbb{P}^3\mathbb{C}$

$\pi \cap \overline{\pi}$

$$\begin{cases} Re(-)=0 \\ Im(-)=0 \end{cases} \to \begin{cases} x-3=0 \\ y-2z=0 \end{cases}$$

$$\begin{cases} x = 0\cdot z+3 \\ y = 2\cdot z+0 \\ z = 1\cdot z \end{cases}$$

$r_\infty = [(0,2,1,0)]$

$$\begin{cases} ix+y-2z-3i=0 \\ -ix+y-2z+3i=0 \end{cases}$$

$$\begin{cases} 2y-4z=0 \\ 2ix-6i=0 \end{cases} \to \begin{cases} x-3=0 \\ y-2z=0 \end{cases}$$

$$\begin{cases} x_1-3x_4=0 \\ x_2-2x_3=0 \end{cases} \quad x_4=0$$

$$x_1 = 0$$
$$x_2 = 2x_3 \rightarrow [(0\ 2\ 1\ 0)]$$

↑ 4 componenti!

$$r = [(3,0\,0);\mathcal{L}((0\,2\,1))]$$

⚠ per i punti impropri è essenziale scrivere le classi di equivalenza → le coord. sono omogenee.

$$[(a\ b\ c\ d)] = \{\lambda(a,b,c,d) \mid \lambda \in \mathbb{K}\}.$$

$$[(x,y,z,1)] \longrightarrow (x,y,z)$$

$\pi: X = 2i$ trovare i punti reali

$\begin{cases} x = 2i \\ x = -2i \end{cases}$ ϕ in $AG(3, \mathbb{R})$ non ce ne sono

in $\mathbb{P}^3\mathbb{R}$ sono la retta di

eq. $\begin{cases} x_4 = 0 \\ x_1 = 0 \end{cases}$

$\begin{cases} x_1 - 2i x_4 = 0 \\ x_1 + 2i x_4 = 0 \end{cases}$ $\rightarrow \begin{matrix} x_4 = 0 \\ x_1 = 0 \end{matrix}$ $\rightarrow \mathcal{L}((0100), (0010))$.

x

Scrivere una matrice $M \in \mathbb{R}^{3,3}$ tale che $\underline{(1\,2\,3)}$ e $\underline{(0\,1\,2)}$ siano autovettori di M di autovalori risp. 1 e 2.

$$P^{-1} M P = D \quad \text{con} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$$M = P D P^{-1}$$

$$M\begin{bmatrix}1\\2\\3\end{bmatrix} = 1 \cdot \begin{bmatrix}1\\2\\3\end{bmatrix} \qquad \begin{cases} m_{11}+2m_{12}+3m_{13} = 1 \\ m_{21}+2m_{22}+3m_{23} = 2 \\ m_{31}+2m_{32}+3m_{33} = 3 \end{cases}$$

$$M\begin{bmatrix}0\\1\\2\end{bmatrix} = 2\begin{bmatrix}0\\1\\2\end{bmatrix} \qquad \begin{cases} 1m_{12}+2m_{13} = 0 \\ 1m_{22}+2m_{23} = 2 \\ 1m_{32}+2m_{33} = 4 \end{cases}$$

$m_{12} = -2m_{13} \rightarrow m_{12}=m_{13}=0$

$m_{22} = 2-2m_{23} \quad m_{23}=0$

$m_{32} = 4-2m_{33} \quad m_{33}=0$

$$\begin{bmatrix}1 & 0 & 0\\ -2 & 2 & 0\\ -5 & 4 & 0\end{bmatrix}$$

$m_{11} = 1$

$m_{21} = 2-4 = -2$

$m_{31} = 3-8 = -5$

Base $W^{\perp}$ ove $W = \{(x_1\, x_2\, x_3\, x_4) \mid x_1 + x_2 - x_3 = 0,\ x_1 + x_3 = 0,\ x_2 - 2x_3 = 0\}$.

$$W = \mathcal{L}\big((1\,1\,-1\,0), (1\,0\,1\,0), (0\,1\,-2\,0)\big)^{\perp}$$

$$W^{\perp\perp} = \mathcal{L}\big((1\,1\,-1\,0), (1\,0\,1\,0), (0\,1\,-2\,0)\big).$$

i vettori sono indip?

$$rk\begin{pmatrix} \boxed{\begin{matrix}1 & 1\\ 1 & 0\end{matrix}} & \begin{matrix}-1 & 0\\ 1 & 0\end{matrix} \\ \cancel{0 \quad 1} & \cancel{-2 \quad 0} \end{pmatrix} = 2$$

dim $W = 2$

$$\mathcal{B} = \big((1\,1\,-1\,0), (1\,0\,1\,0)\big).$$

'Sistema lin. omogeneo   2 eq. 4 incognite

$(1\ -1\ 0\ 1) \in S.$

$$\begin{cases} 0=0 \\ 0=0 \end{cases} \quad \infty^4$$

$\underline{\infty^2}$

$$(+1\ -1\ 0\ 1)^{\perp} = \mathcal{L}((1\ 1\ 0\ 0), (0\ 1\ 0\ 1), (0\ 0\ 1\ 0)).$$

$$\mathcal{L}((1\ -1\ 0\ 1)) = \{(x_1\ x_2\ x_3\ x_4) : x_1 + x_2 = 0,\ x_2 + x_4 = 0,\ x_3 = 0\}.$$

$$\begin{cases} x_1 + x_2 = 0 \\ x_3 = 0 \end{cases} \qquad \begin{cases} x_3 = 0 \\ x_1 + x_2 = 0 \end{cases}$$

Un sistema lineare di 4 eq. in 2 incognite non omogeneo che abbia ∞^3 soluzioni e tale che $(1\,1\,0\,1)$ sia sol.

1) il sistema deve avere rango $= 1$ $\Rightarrow$ 2 eq. proporzionali

2) il sistema non è omogeneo.

$\Downarrow$

Una eq. soddisfatta da $(1\,1\,0\,1)$ che non sia omogenea.

+ ~~un~~ suo multiplo.

$$\begin{cases} x_1 = 1 \\ 2x_1 = 2 \end{cases} \qquad \begin{cases} x_1 + x_2 + x_3 + x_4 = 3 \\ 2 \ldots \end{cases}$$

Si scriva una matrice M 4×4 che abbia 2 e 3 fra i suoi autovalori con $m_g(2)=1$ e $m_g(3)=2$ con M <u>non</u> diagonalizzabile.

$\exists$ autovalore con $m_g \neq m_a$

se $\exists\,\lambda$ autovalore di M con $\lambda \neq 2,3 \Rightarrow m_a(\lambda)=m_g(\lambda)=1$

$\Rightarrow M$ diagonalizzabile.

- $m_g(2)=1 \quad m_a(2)=2$
- $m_g(3)=2 \quad m_a(3)=3$

$$\begin{bmatrix} 2 & 1 & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & & & \\ & 3 & & \\ & & 3 & 1 \\ & & 0 & 3 \end{bmatrix}$$

$M \in \mathbb{R}^{4,4}$ non diagonale con autovalori 2 e 3 e

$m_g(2) = 1, \quad m_g(3) = 2$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 5 \\ \hline 0 & 0 & 0 & 4 \end{array}\right] \leftarrow \text{diagonalizzabile}$$

autovalori <u>solo</u> 2 e 3

$\Rightarrow$ si ha necessariamente una matrice non diagonalizzabile.

$\Rightarrow \sum m_g(\lambda) = 3 < 4$

$$\left[\begin{array}{ccc|c} 2 & & & 0 \\ & 3 & & 0 \\ & & 3 & 1 \\ \hline & & & 2 \end{array}\right]$$

piano reale che contenga

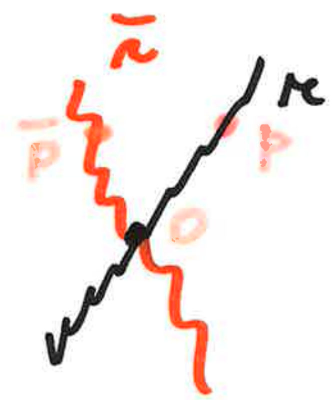
$$r\begin{cases} x+iy=0 \\ x-iy+z=0 \end{cases}$$

Il piano $\exists$. $\qquad (0\,0\,0)\in r. \qquad [(0\,0\,0\,1)]$

$$2x+z=0$$

piano per 2 punti della retta
(di cui uno reale) + coniugato dell'altro p.to.

$y=1 \quad x=-i \qquad -2i+z=0 \quad z=2i$



pr. per $\quad [(0\,0\,0\,1)]$

$[(-i,1,2i,1)]=P\in r$

$[(i,1,-2i,1)]=\bar{P}\in\bar{r}$

$$\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ 0 & 0 & 0 & 1 \\ -i & 1 & 2i & 1 \\ i & 1 & -2i & 1 \end{vmatrix} = 0$$

$$0 = x_1 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 2i & 1 \\ 1 & -2i & 1 \end{vmatrix} + x_3 \begin{vmatrix} 0 & 0 & 1 \\ -i & 1 & 1 \\ i & 1 & 1 \end{vmatrix} + x_2 \begin{vmatrix} 0 & 0 & 1 \\ -i & 2i & 1 \\ i & -2i & 1 \end{vmatrix}$$

$$-4i\, x_1 - 2i\, x_3 = 0$$

$$-4i\, x - 2i\, \cancel{z} = 0$$

$$\Downarrow$$

$$2x + z = 0$$

$B\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ è soddisfatto.

$$\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{pmatrix}\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$a_{11}+2a_{12}=0$$

$$a_{21}+2a_{22}=0$$

$$\begin{pmatrix}-2a_{12} & a_{12} & a_{13}\\ -2a_{22} & a_{22} & a_{23}\end{pmatrix}$$

$$\rightarrow \underbrace{\left(\underset{\bar{v}_1}{\begin{pmatrix}-2 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}}, \underset{\bar{v}_2}{\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}}, \underset{\bar{v}_3}{\begin{pmatrix}0 & 0 & 0\\ -2 & 1 & 0\end{pmatrix}}, \underset{\bar{v}_4}{\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}}\right)}_{=\mathcal{B}}$$

$$-1\cdot\bar{v}_1 + 0\cdot\bar{v}_2 + 1\bar{v}_3 + 2\bar{v}_4 = B$$

$$(-1, 0, 1, 2)$$

ranghi el. di $V = \{0, 1, 2\}$.

Sia W sp. vett. generato da

$$\left(\begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \right)$$

possibili ranghi el. di $W \rightarrow \{0, 2\}$.

$$\left(\alpha \begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \beta \begin{pmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \right)$$

$\beta \neq 0 \Rightarrow rk = 2$

$\beta = 0, \alpha \neq 0 \Rightarrow rk = 2$

$\alpha = \beta = 0 \rightarrow rk = 0$

B)

$$\begin{pmatrix} 1 & k & 0 \\ k & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ k \\ 1 \end{pmatrix} \qquad k=0,\ \lambda=1$$

$$rk\begin{pmatrix} 1 & 1 \\ k & 0 \\ 1 & 1 \end{pmatrix} = 1 \quad \Leftrightarrow k=0$$

D)

$$\left[\begin{array}{ccc|c} 1 & k+1 & 2 & 0 \\ 1 & 0 & 3+k & k+3 \\ 1 & 1 & 0 & -2 \end{array}\right] \qquad rk(A) \geq 2$$

$$\det(A) = \begin{vmatrix} k+1 & 2 \\ 0 & k+3 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 1 & k+3 \end{vmatrix} =$$

$$= (k+3)(k+1) - (k+3) + 2$$

$= k^2 + 3k + 2$

$k = -1$ oppure $k = -2$

$k \neq -1, -2 \Rightarrow rk(A) = rk(A|B) = 3 \Rightarrow \exists!$ sol.

$k = -1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 1 & 0 & 2 & 2 \\ 1 & 1 & 0 & -2 \end{array}\right] \quad rk(A|B) = 3 \Rightarrow \nexists \text{ sol.}$$

$k = -2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & -2 \end{array}\right] \quad rk(A|B) = 3$$

$1 \ 1 \ 0 = 2(1 \ 0 \ 1) - (1 \ -1 \ 2)$

U, V sono in somma diretta.

$U \cap V = \{0\}$.

$$U \cap V = \left\{ f \in \mathbb{C}[x]_{\leq 3} : \begin{array}{l} f(k)=0 \\ f(0)=0 \\ f(i)=0 \\ f(1)=0 \end{array} \right\}.$$

sono polinomi che hanno radici $0, k, i, 1$

$\Rightarrow$ se $k \notin \{0, i, 1\} \Rightarrow 4$ radici distinte, $\deg \leq 3 \Rightarrow f(x) \equiv 0$.

$\Rightarrow$ somma diretta.

$\Rightarrow k \in \{0, i, 1\} \Rightarrow \quad x(x-i)(x-1) \in U \cap V \Rightarrow$

somma non diretta.

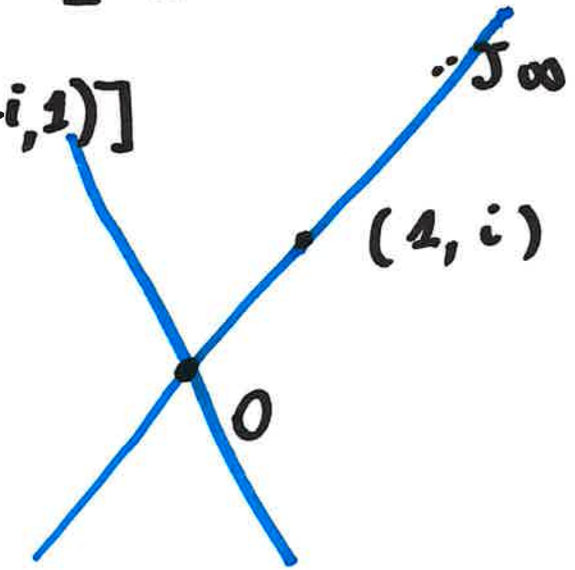
F) $[(0\ 0\ i)] = [(0\ 0\ 1)] \sim (0\ 0)$

$[(1\ i\ 0)] = J_\infty \quad \to [(1, -i, 0)] = \bar{J}_\infty$] circ. generalizzata

$P = [(1\ i\ 1)] \quad \sim (1, i)$

$y = ix$

$\bar{P} = [(1, -i, 1)]$



$x^2 + y^2 = 0$

G) proiezione ort. di $P = (2,2,4)$ su $x - 2z = 0$

$\downarrow$

normale $(1,0,-2) = \bar{n}$

$$\frac{x-2}{1} = \frac{y-2}{0} = \frac{z-4}{-2}$$

$$\begin{cases} y = 2 \\ -2x + 4 - z + 4 = 0 \\ x - 2z = 0 \end{cases}$$

$$\begin{cases} y = 2 \\ 2x + z - 8 = 0 \\ x - 2z = 0 \end{cases} \qquad 5z = 8$$

$$z = \frac{8}{5}$$

$$x = \frac{16}{5}$$

A) ATT. A QUANDO LA CONICA È RIDUCIBILE!

$$A = \begin{pmatrix} 1 & K & 0 \\ K & 0 & K+1 \\ 0 & K+1 & 1 \end{pmatrix}$$

$\det A^* = K^2 \geq 0$

$++$ oppure

$+0$ per $K=0$

per $K=0$ $\quad \det A = \det \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \neq 0$

parabola.

$\det A = -(K+1)^2 - K^2 \neq 0 \ \forall K$: $K \neq 0$ <u>ellisse</u>.

$\dim W^{\perp}$ ove $W = \mathcal{L}(S)$ con S soluzioni del sistema.

se il sistema non è omogeneo e

$S \neq \emptyset \Rightarrow \dim \mathcal{L}(S) = n - r + 1$

ove $n = 3$ $\quad r =$ rango.

$$\dim \mathcal{L}(S)^{\perp} = n - (n - r + 1) = r - 1$$

se $S = \emptyset \Rightarrow \mathcal{L}(S) = \{\underline{0}\} \Rightarrow \dim \mathcal{L}(S)^{\perp} = n$

$$\left[\begin{array}{ccc|c} 1 & 2 & k & 0 \\ 1 & 0 & -1 & k \\ 0 & 2 & 4 & 3 \end{array}\right] \qquad \det(A) = \det \begin{bmatrix} 1 & 2 & k \\ 0 & -2 & -1-k \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{vmatrix} -2 & -1-k \\ 2 & 4 \end{vmatrix} = -8 + 2 + 2k = 2k - 6$$

$k \neq 3 \Rightarrow \exists!$ soluzione $\Rightarrow$

$\dim \mathcal{L}(S) = 1 \quad \dim S^{\perp} = 2$

$k = 3$

$$rk \begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 0 & -1 & 3 \\ 0 & 2 & 4 & 3 \end{bmatrix} =$$

$$\det \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 3 \\ 0 & 2 & 3 \end{pmatrix} = -6 - 2 \cdot 3 = -12 \neq 0$$

$\Rightarrow S = \emptyset \Rightarrow \dim \mathcal{L}(S)^{\perp} = 3.$

D)

$$\left[\begin{array}{cc|ccc} 1 & 1 & & 0 & \\ & 1 & & & \\ \hline & & 3 & & \\ & 0 & & 3 & 1 \\ & & & & 3 \end{array}\right]$$

auf 1, 3

$rk(A-I)=4$

$m_g(1)=1$

$m_a(1)=2$

$m_g(3)=2$

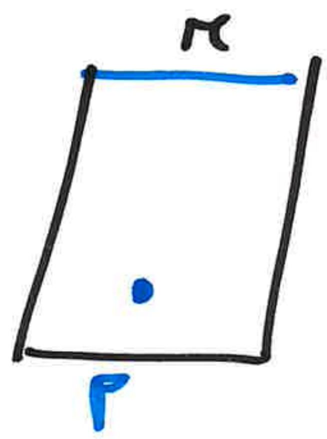
$m_a(3)=3$

E) $\mathcal{B}= \left(\underset{\bar{e}_1}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}, \underset{\bar{e}_2}{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}, \underset{\bar{e}_3}{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}} \right)$

$\bar{v} = \bar{e}_1 + 3\bar{e}_2 + 0\bar{e}_3 \qquad (1,3,0)$

F) retta per $(1\,0\,0)$ e incidente ort.
$P_{//}$ $(x+y-1) = 0 = z$

oss: $\boxed{P \in r}$



1° piano → piano del fascio per P

2° piano → piano per P con normale // r

c) posizione reciproca di 3 piani

$$\begin{array}{ccccc} \pi_K & 1 & K & 2i & 0 \\ \vartheta_K & 1 & -K & 2K & 0 \\ \sigma_K & 1 & 1 & -1 & 4 \end{array}$$

π_K rispetto ϑ_K $rk\begin{bmatrix} 1 & K & 2i \\ 1 & -K & 2K \end{bmatrix}$ $\quad$ $K \neq 0$ $\;$ $rk = 2$

$K = 0$ $\;$ $rk = 2$

π_K e ϑ_K si intersecano sempre in una retta propria $\quad (0\,0\,0) \in \pi_K \cap \vartheta_K$

$\pi_K \cap \sigma_K$ $\quad \begin{bmatrix} 1 & K & 2i \\ 1 & 1 & -1 \end{bmatrix}$ $\quad$ $K \neq 1$ $\;$ $rk = 2$

$K = 1$ $\;$ $rk = 2$

si intersecano in una retta propria.

$$rk\begin{pmatrix} -1 & -k & 2k & 0 \\ 1 & 1 & -1 & 4 \end{pmatrix}$$

$k \neq -1$

$rk = 2$

$k = -1$

$$\begin{matrix} 1 & 1 & -2 & 0 \\ 1 & 1 & -1 & 4 \end{matrix}$$

$\rightarrow$ retta propria.

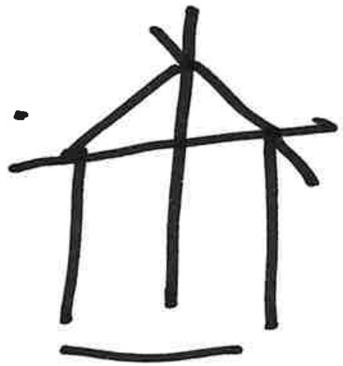
a 2 a 2 i piani si intersecano in 3 rette proprie.

$$rk\begin{bmatrix} 1 & k & 2i \\ 1 & -k & 2k \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & k & 2i \\ 0 & -2k & 2k-2i \\ 0 & 1-k & -1-2i \end{bmatrix}$$

(*) $\det \begin{bmatrix} -2k & 2k-2i \\ 1-k & -1-2i \end{bmatrix} = 2k(1+2i)+(k-1)(2k-2i)$

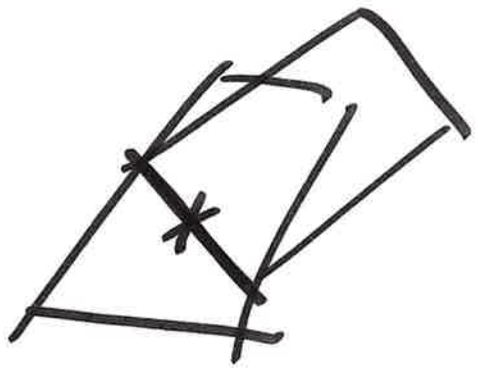
per soluzioni di * i 3

piani hanno un pto improprio in comune.
(mentre rk (A|B) é 3).



k non soluzione di (*) $\Rightarrow$ le 3 rette non sono //.

3 piani hanno sempre 1 pto in comune.



considerciamo la

retta reale contenuta

nel piano

π_K e la intersechiamo

col piano $x+y-z+4=0$

↓

<u>pto reale</u>

guardiamo se tale punto

sta anche su π_K

osservando che essendo un pto reale

Ca ma $x \in \mathbb{R}$ e quindi anche

$-ky + 2kz \in \mathbb{R}$

con $y, z \in \mathbb{R}$

e dunque anche $k \in \mathbb{R}$.

$$rk \begin{bmatrix} 1 & 2 & k & 0 \\ 1 & 0 & 1 & k \\ 0 & 2 & 4 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & k & 0 \\ 0 & -2 & -k+1 & k \\ 0 & 2 & 4 & 3 \end{bmatrix} \sim$$

$k = -5 \quad rk(A) = 2$

$$\begin{bmatrix} 1 & 2 & -5 & 0 \\ 0 & -2 & -4 & -5 \\ 0 & 2 & 4 & 3 \end{bmatrix}$$

$rk(A) = 2 \quad rk(A|B) = 3$

$-2y - 4z = 5$

$2y + 4z = 3$

$S = \emptyset \Rightarrow S^{\perp} = \mathbb{R}^3 \Rightarrow \dim = 3$