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Corrigendum: Face-magic labelings of type (a, b, c) from edge-magic labelings of type (α, β)

BRYAN J. FREYBERG

DEPARTMENT OF MATHEMATICS & STATISTICS UNIVERSITY OF MINNESOTA DULUTH, DULUTH MN, USA frey0031@d.umn.edu

Abstract. In the paper, Face-magic labelings of type (a, b, c) from edgemagic labelings of type (α, β) , Bull. Inst. Combin. Appl. **93** (2021), 80 – 102, the author made a typographical error in the statement of Theorem 5.4 and overlooked the trivial case of s = r = 0 in a subcase of Theorems 5.11 and 5.16. We correct the three theorem statements, and prove a stronger non-existence result which provides the correction of Theorems 5.11 and 5.16.

In the original paper [1], the author made a typographical error involving two of the exceptions $((a, b, c) \in \{(0, 1, 0), (0, 1, 1)\})$ in the statement of Theorem 5.4. We give the corrected statement below. The proof given in [1] remains correct.

Theorem 5.4. For any integers $a, b, c \in \{0, 1\}$, $n, k \ge 3$, the kC_n -cycle admits a face-magic labeling of type (a, b, c), except in the following cases.

- a = b = 0.
- a = c = 0, b = 1; n is odd and k is even.
- a = 0, b = c = 1; n and k are both even.

Secondly, the author overlooked the trivial subcase s = r = 0 when (a, b, c) = (1, 0, 0) in the statement and proof of Theorems 5.11 and 5.16. This subcase corresponds to a type (1, 0, 0) face-magic labeling of the fan F_n and wheel

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 W_n , respectively. We provide the corrected theorem statements next and then prove the corrected statements by showing a stronger result; that no graph which contains $K_4 - \{e\}$ as a subgraph admits a face-magic labeling of type (1, 0, 0).

Theorem 5.11. Let $a, b, c \in \{0, 1\}$ and $r, s \ge 0$. The subdivided fan $F_n(r, s)$ admits a face-magic labeling of type (a, b, c) for any $n \ge 3$ unless a = b = 0, or (a, b, c) = (1, 0, 0) and s = r = 0.

Theorem 5.16. Let $a, b, c \in \{0, 1\}$ and $r, s \ge 0$. The subdivided wheel $W_n(r, s)$ admits a face-magic labeling of type (a, b, c) for any $n \ge 3$ unless a = b = 0 or (a, b, c) = (1, 0, 0) and s = r = 0.

Theorem. If G is a graph and it contains $K_4 - \{e\}$ as a subgraph, then G does not admit a face-magic labeling of type (1,0,0).

Proof. Suppose to the contrary G admits such a labeling f where the weight of every 3-sided face of G is k. Let $H \cong K_4 - \{e\}$ be a subgraph of G with $V(H) = \{x_1, x_2, y_1, y_2\}$ where x_1 and x_2 are the vertices of degree 3. Then the weight of the face formed by the cycle x_1, y_1, x_2, x_1 is $f(x_1) + f(y_1) + f(x_2)$, and the weight of the face formed by the cycle x_1, y_2, x_2, x_1 is $f(x_1) + f(y_2) + f(x_2)$. Therefore, $f(y_1) = f(y_2)$ since the two weights equal k. But this contradicts the fact that f is a bijection, so we have proved the claim.

As an immediate corollary, we have the following.

Corollary. Let $n \ge 3$. If $G \cong W_n$ or $G \cong F_n$, then G does not admit a face-magic labeling of type (1, 0, 0).

References

[1] Freyberg, B. Face-magic labelings of type (a, b, c) from edge-magic labelings of type (a, b, c), Bull. Inst. Combin. Appl., **93** (2020) 80–102.