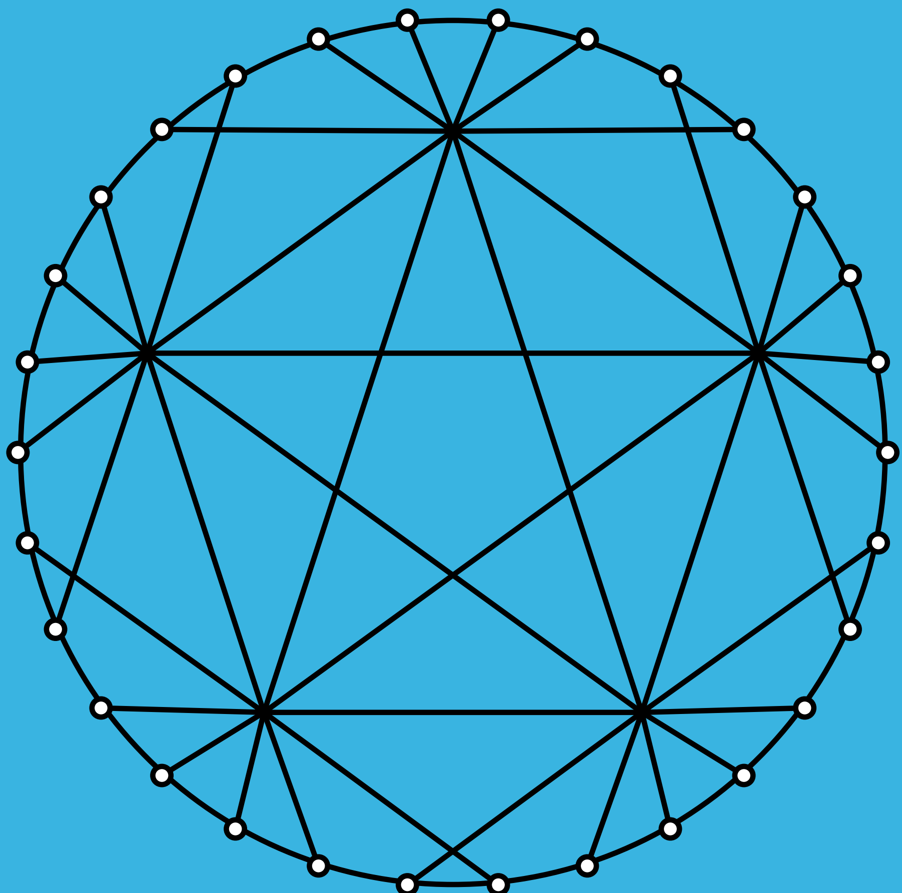
BULLETIN of The INSTITUTE of COMBINATORICS and its APPLICATIONS

Volume 87
October 2019



Boca Raton, FL, U.S.A.

ISSN 1182 - 1278

BULLETIN OF THE ICA
Volume **87** (2019), Pages 103–113

Transitive distance-regular graphs from the unitary groups U(3, 4), U(4, 3) and U(5, 2)

Andrea Švob

University of Rijeka, Rijeka, Croatia
asvob@math.uniri.hr

Abstract: In this paper we classify distance-regular graphs, including strongly regular graphs, admitting a transitive action of the unitary groups $\mathrm{U}(3,4)$, $\mathrm{U}(4,3)$ and $\mathrm{U}(5,2)$ for which the rank of the permutation representation is at most 15. We give details about the constructed graphs. The graphs constructed in the paper were known before, but have been obtained in a different way.

1 Introduction

We assume that the reader is familiar with the basic facts of the group theory, the theory of strongly regular graphs and the theory of distance-regular graphs. We refer the reader to [6, 17] for relevant background reading in the group theory, to [3, 18] for the theory of strongly regular graphs, and to [4, 13] for the theory of distance-regular graphs.

A construction of distance-regular graphs (DRGs), and especially strongly regular graphs (SRGs), from finite groups gave an important contribution to the graph theory and the design theory (see [3, 4]). Recently, in [11, 12] the authors found new SRGs admitting a transitive action of some finite

AMS (MOS) Subject Classifications: 05E30, 05E18.
Key words and phrases: strongly regular graph, distance-regular graph, unitary group.

Received: 9 September 2019
Accepted: 12 September 2019

simple groups and show how one can use groups as a tool to produce wide range of interesting regular graphs.

In this paper we classify distance-regular graphs, including strongly regular graphs, admitting a transitive action of the unitary groups $\mathrm{U}(3,4)$, $\mathrm{U}(4,3)$ and $\mathrm{U}(5,2)$ for which the rank of the permutation representation is at most 15 (i.e. the number of orbits of the stabilizer of a vertex is at most 15). The graphs constructed in the paper were known before, but have been obtained in a different way. The method of construction is outlined in Section 3. We give a details about the obtained graphs. Note that primitive strongly regular graphs from the groups $\mathrm{U}(3,4)$, $\mathrm{U}(4,3)$ and $\mathrm{U}(5,2)$ are described in [7], [9] and [8]. We refer the reader to [6, 20] for more details about these unitary groups.

We used programmes written for Magma [2] and GAP [14]. The constructed SRGs and DRGs can be found at the link:
http://www.math.uniri.hr/~asvob/DRGs_UniGps.zip.

2 Preliminaries

In this section we define coherent configurations and association schemes, which are the tools for the construction of graphs presented in this paper. We also give basic definitions and properties of DRGs and SRGs.

Definition 2.1 *A coherent configuration on a finite non-empty set Ω is an ordered pair $(\Omega, \mathcal{R})$ with $\mathcal{R} = \{R_0, R_1, \ldots, R_d\}$ a set of non-empty relations on Ω, such that the following axioms hold.*

(i) *There exists t such that $\sum_{i=0}^{t} R_i$ is the identity relation, where $\{R_0, R_1, \ldots, R_t\} \subseteq \{R_0, R_1, \ldots, R_d\}$.*

(ii) *$\mathcal{R}$ is a partition of Ω^2.*

(iii) *For every relation $R_i \in \mathcal{R}$, its converse $R_i^T = \{(y,x) : (x,y) \in R_i\}$ is in $\mathcal{R}$.*

(iv) *There are constants p_{ij}^k known as the intersection numbers of the coherent configuration $\mathcal{R}$, such that for $(x,y) \in R_k$, the number of elements z in Ω for which $(x,z) \in R_i$ and $(z,y) \in R_j$ equals p_{ij}^k.*

We say that a coherent configuration is homogeneous if it contains the identity relation, i.e., if $R_0 = I$. If $\mathcal{R}$ is a set of symmetric relations on Ω, then a coherent configuration is called symmetric. A symmetric coherent configuration is homogeneous (see [5]). Symmetric coherent configurations are introduced by Bose and Shimamoto in [1] and called association schemes. An association scheme with relations $\{R_0, R_1, \ldots, R_d\}$ is called a d-class association scheme.

Let Γ be a graph with diameter d, and let $\delta(u, v)$ denote the distance between vertices u and v of Γ. The ith-neighborhood of a vertex v is the set $\Gamma_i(v) = \{w : \delta(v, w) = i\}$. Similarly, we define Γ_i to be the ith-distance graph of Γ, that is, the vertex set of Γ_i is the same as for Γ, with adjacency in Γ_i defined by the ith distance relation in Γ. We say that Γ is distance-regular if the distance relations of Γ give the relations of a d-class association scheme, that is, for every choice of $0 \le i, j, k \le d$, all vertices v and w with $\delta(v, w) = k$ satisfy $|\Gamma_i(v) \cap \Gamma_j(w)| = p^k_{ij}$ for some constant p^k_{ij}. In a distance-regular graph, we have that $p^k_{ij} = 0$ whenever $i + j < k$ or $k < |i - j|$. A distance-regular graph Γ is necessarily regular with degree p^0_{11}; more generally, each distance graph Γ_i is regular with degree $k_i = p^0_{ii}$.

An equivalent definition of distance-regular graphs is the existence of the constants $b_i = p^i_{i+1,1}$ and $c_i = p^i_{i-1,1}$ for $0 \le i \le d$ (notice that $b_d = c_0 = 0$). The sequence $\{b_0, b_1, \ldots, b_{d-1}; c_1, c_2, \ldots, c_d\}$, where d is the diameter of Γ is called the intersection array of Γ. Clearly, $b_0 = k$, $b_d = c_0 = 0$, $c_1 = 0$.

A regular graph is strongly regular with parameters (v, k, λ, μ) if it has v vertices, degree k, and if any two adjacent vertices are together adjacent to λ vertices, while any two non-adjacent vertices are together adjacent to μ vertices. A strongly regular graph with parameters (v, k, λ, μ) is usually denoted by $\mathrm{SRG}(v, k, \lambda, \mu)$. A strongly regular graph is a distance-regular graph with diameter 2 whenever $\mu \neq 0$. The intersection array of an SRG is given by $\{k, k - 1 - \lambda; 1, \mu\}$.

3 SRGs and DRGs constructed from the groups

Let G be a finite permutation group acting on the finite set Ω. This action induce the action of the group G on the set $\Omega \times \Omega$. For more information see [19]. The orbits of this action are the sets of the form $\{(\alpha g, \beta g) : g \in G\}$. If G is transitive, then $\{(\alpha, \alpha) : \alpha \in \Omega\}$ is one such orbit. If the rank of G is r, then it has r orbits on $\Omega \times \Omega$. Let $|\Omega| = n$ and Δ_i is one of these orbits. We say that the $n \times n$ matrix A_i, with rows and columns indexed by Ω and entries

$$A_i(\alpha, \beta) = \begin{cases} 1, & \text{if } (\alpha, \beta) \in \Delta_i \\ 0, & \text{otherwise.} \end{cases}$$

is called the adjacency matrix for the orbit Δ_i.

Let $A_0, \ldots, A_{r-1}$ be the adjacency matrices for the orbits of G on $\Omega \times \Omega$. These satisfy the following conditions.

(i) $A_0 = I$, if G is transitive on Ω. If G has s orbits on Ω, then I is a sum of s adjacency matrices.

(ii) $\sum_i A_i = J$, where J is the all-one matrix.

(iii) If A_i is an adjacency matrix, then so is its transpose A_i^T.

(iv) If A_i and A_j are adjacency matrices, then their product is an integer-linear-combination of adjacency matrices.

If A_i is symmetric, then the corresponding orbit is called self-paired. Further, if $A_i = A_j^T$, then the corresponding orbits are called mutually paired.

The graphs obtained in this paper are constructed using the method described in [10] which can be rewritten in terms of coherent configurations as shown in [12].

Theorem 3.1 [12, Theorem 1] *Let G be a finite permutation group acting transitively on the set Ω and $A_0, \ldots, A_d$ be the adjacency matrices for orbits of G on $\Omega \times \Omega$. Let $\{B_1, \ldots, B_t\} \subseteq \{A_1, \ldots, A_d\}$ be a set of adjacency matrices for the self-paired or mutually paired orbits. Then $M = \sum_{i=1}^{t} B_i$ is the adjacency matrix of a regular graph Γ. The group G acts transitively on the set of vertices of the graph Γ.*

Using this method one can construct all regular graphs admitting a transitive action of the group G. We will be interested only in those regular graphs that are distance-regular, and especially strongly regular.

The running time complexity of the algorithm used for the construction of the graphs depends on a number of parameters, such as the size of the used group and subgroup, the number of orbits of the stabilizer and the number of self-paired and mutually paired orbits in a particular case.

3.1 SRGs and DRGs from the group U(3, 4)

In [7] the authors constructed primitive strongly regular graphs by defining incidence structures on conjugacy classes of maximal subgroups of the simple group $\mathrm{U}(3,4)$. Here, we give a classification of all transitive (not just primitive) SRGs and DRGs of diameter $d \geq 3$ for which the number of orbits of the vertex stabilizer is at most 15.

The group $\mathrm{U}(3,4)$ is the simple group of order 62400. Up to conjugation it has 34 subgroups, 6 of them have rank at most 15.

In Table 1 we give the list of all the subgroups (for which the rank of the permutation representation is at most 15) $H_i^1 \leq \mathrm{U}(3,4)$ which lead to the construction of SRGs or DRGs of diameter $d \geq 3$.

Subgroup	Structure	Order	Index	Rank	Primitive
H_1^1	$(E_4.E_{16}):Z_5$	320	195	6	no
H_2^1	$Z_5 \times A_5$	300	208	5	yes
H_3^1	$(E_4.E_{16}):Z_3$	192	325	10	no
H_4^1	$(E_{25}:Z_3):Z_2$	150	416	9	yes

Table 1: Subgroups of the group $\mathrm{U}(3,4)$ that give rise to DRGs.

Using the method described in Theorem 3.1 we obtained all DRGs on which the group $\mathrm{U}(3,4)$ acts transitively and for which the rank of the permutation representation of the group is at most 15.

Theorem 3.2 *Up to isomorphism there are exactly three strongly regular graphs and exactly three distance-regular graphs of diameter* $d \geq 3$ *admitting an transitive action of the group* $\mathrm{U}(3,4)$*, having the rank at most 15. The constructed SRGs have parameters* $(208, 75, 30, 25)$*,* $(325, 68, 3, 17)$ *and* $(416, 100, 36, 20)$*, and the DRGs of diameter* $d \geq 3$ *have* 195*,* 208*,* 325 *vertices, respectively. Details about the obtained strongly regular graphs are given in Table 2 and details about the obtained DRGs with* $d \geq 3$ *are given in Table 3.*

Graph Γ	Parameters	$Aut(\Gamma)$
$\Gamma_1^1 = \Gamma(\mathrm{U}(3,4), H_2^1)$	(208,75,30,25)	$\mathrm{U}(3,4) : Z_4$
$\Gamma_2^1 = \Gamma(\mathrm{U}(3,4), H_3^1)$	(325,68,3,17)	$\mathrm{U}(4,4) : Z_4$
$\Gamma_3^1 = \Gamma(\mathrm{U}(3,4), H_4^1)$	(416,100,36,20)	$G(2,4) : Z_2$

Table 2: SRGs constructed from the group $\mathrm{U}(3,4)$.

Graph Γ	Vertices	Diameter	Intersection array	$Aut(\Gamma)$
$\Gamma_4^1 = \Gamma(\mathrm{U}(3,4), H_1^1)$	195	3	$\{64, 42, 1; 1, 21, 64\}$	$\mathrm{U}(3,4) : Z_4$
$\Gamma_5^1 = \Gamma(\mathrm{U}(3,4), H_2^1)$	208	3	$\{12, 10, 5; 1, 1, 8\}$	$\mathrm{U}(3,4) : Z_4$
$\Gamma_6^1 = \Gamma(\mathrm{U}(3,4), H_3^1)$	325	3	$\{64, 60, 1; 1, 15, 64\}$	$(Z_5 \times \mathrm{U}(3,4)) : Z_4$

Table 3: DRGs constructed from the group $\mathrm{U}(3,4)$, $d \geq 3$.

Remark 3.3 *The strongly regular graphs Γ_1^1 and Γ_3^1 are isomorphic to the SRGs constructed in [7]. Moreover, Γ_3^1 is known as $G(2,4)$ graph. It is locally the Janko graph, and the second subconstituent of the Suzuki tower (see [16]). Further, the strongly regular graphs Γ_2^1 and Γ_3^1 are rank 3 graphs. The SRG Γ_2^1 is isomorphic to the graph that belongs to the family of* $\mathrm{U}(4,4)$ *graphs described in [15] and constructed in the following way. Let V be an 4-dimensional vector space over the field $GF(4)$, provided with a nondegenerate Hermitean form. The vertices of the graph are the isotropic points that are adjacent when orthogonal.*

Remark 3.4 *The distance-regular graphs Γ_4^1 and Γ_6^1 have diameter* 3. *They belong to the family of antipodal but not bipartite distance regular graphs. The graph Γ_5^1 having 208 vertices and diameter* 3 *is known as unitary non-isotropic graph. See [4] for more information.*

3.2 SRGs and DRGs from the group $\mathrm{U}(4,3)$

In [9] the authors constructed primitive strongly regular graphs by defining incidence structures on conjugacy classes of maximal subgroups of the simple group $\mathrm{U}(4,3)$. Here, we give a classification of all transitive (not just primitive) SRGs and DRGs of diameter $d \geq 3$ for which the number of orbits of the vertex stabilizer is at most 15.

The group $\mathrm{U}(4,3)$ is the simple group of order 3265920. Up to conjugation it has 381 subgroups, 27 of them have rank at most 15.

In Table 4 we give the list of all the subgroups (for which the rank of the permutation representation is at most 15) $H_i^1 \leq \mathrm{U}(4,3)$ which lead to the construction of SRGs or DRGs of diameter $d \geq 3$.

Subgroup	Structure	Order	Index	Rank	Primitive
H_1^2	$E_{81} : A_6$	29160	112	3	yes
H_2^2	$\mathrm{U}(4,2)$	25920	126	3	yes
H_3^2	$L(3,4)$	20160	162	3	yes
H_4^2	$3_+^{1+4} \times 2S_4$	11664	280	3	yes
H_5^2	$\mathrm{U}(3,3)$	6048	540	4	yes
H_6^2	$E_{81} : A_5$	4860	672	6	no

Table 4: Subgroups of the group $\mathrm{U}(4,3)$ that give rise to DRGs.

Using the method described in Theorem 3.1 we obtained all DRGs for which the rank of the permutation representation of the group is at most 15.

Theorem 3.5 *Up to isomorphism there are exactly six strongly regular graphs and there are no distance-regular graphs of diameter* $d \geq 3$ *admitting an transitive action of the group* $\mathrm{U}(4,3)$, *having the rank at most 15. The SRGs have parameters* $(112, 30, 2, 10)$, $(126, 45, 12, 18)$, $(162, 56, 10, 24)$, $(280, 36, 8, 4)$, $(540, 224, 88, 96)$ *and* $(672, 176, 40, 48)$. *Details about the obtained strongly regular graphs are given in Table 5.*

Graph Γ	Parameters	$Aut(\Gamma)$
$\Gamma_1^2 = \Gamma(\mathrm{U}(4,3), H_1^2)$	(112,30,2,10	$\mathrm{U}(4,3) : D_8$
$\Gamma_2^2 = \Gamma(\mathrm{U}(4,3), H_2^2)$	(126,45,12,18)	$(\mathrm{U}(4,3) : Z_2) : Z_2$
$\Gamma_3^2 = \Gamma(\mathrm{U}(4,3), H_3^2)$	(162,56,10,24)	$(\mathrm{U}(4,3) : Z_2) : Z_2$
$\Gamma_4^2 = \Gamma(\mathrm{U}(4,3), H_4^2)$	(280,36,8,4)	$\mathrm{U}(4,3) : D_8$
$\Gamma_5^2 = \Gamma(\mathrm{U}(4,3), H_5^2)$	(540,224,88,96)	$\mathrm{U}(4,3) : D_8$
$\Gamma_6^2 = \Gamma(\mathrm{U}(4,3), H_6^2)$	(672,176,40,48)	$\mathrm{U}(6,2) : S_3$

Table 5: SRGs constructed from the group $\mathrm{U}(4,3)$.

Remark 3.6 *The strongly regular graphs* Γ_1^2, Γ_2^2, Γ_3^2 *and* Γ_4^2 *are rank 3 graphs and are isomorphic to the SRGs constructed in [9]. The graph* Γ_5^2 *is known as* $\mathrm{U}(4,3)$ *graph and is isomorphic to the graph constructed in [11]. Further, the strongly regular graph* Γ_6^2 *is known as* $\mathrm{U}(6,2)$ *graph. Both* Γ_5^2 *and* Γ_6^2 *belong to the family of graphs described in [15] and can be constructed in the following way. Let* V *be an 4-dimensional and 6-dimensional vector space over the field* $GF(16)$ *and* $GF(36)$, *respectively, provided with a nondegenerate Hermitean form. The vertices of the graph are the nonisotropic points, adjacent when joined by a tangent.*

3.3 SRGs and DRGs from the group U(5, 2)

In [8] the authors constructed primitive strongly regular graphs by defining incidence structures on conjugacy classes of maximal subgroups of the simple group U(5, 2). Here, we give a classification of all transitive (not just primitive) SRGs and DRGs of diameter $d \geq 3$ for which the number of orbits of the vertex stabilizer is at most 15.

The group U(5, 2) is the simple group of order 13685760. Up to conjugation it has 556 subgroups, 19 of them have rank at most 15.

In Table 6 we give the list of all the subgroups (for which the rank of the permutation representation is at most 15) $H_i^1 \leq \mathrm{U}(5,2)$ which lead to the construction of SRGs or DRGs of diameter $d \geq 3$.

Subgroup	Structure	Order	Index	Rank	Primitive
H_1^3	$(E_{64} : Z_2).(E_9 : Z_3).SL(2,3)$	829444	165	3	yes
H_2^3	$Z_3 \times \mathrm{U}(4,2)$	46080	176	3	yes
H_3^3	$(E_{16} : E_{16}) : (Z_3 \times A_5)$	77760	297	3	yes
H_4^3	$(((Z_2 \times D_8) : Z_2).A_4).(E_9 : Q_8)$	27648	495	7	no
H_5^3	$O(5,3)$	25920	528	7	no
H_6^3	$((E_4 \times Q_8) : Q_8) : A_5$	15360	891	9	no
H_7^3	$E_{81} : S_5$	9720	1408	7	yes
H_8^3	$(((E_4 \times Q_8) : Q_8) : Z_5) : S_3$	7680	1782	7	no

Table 6: Subgroups of the group U(5, 2) that give rise to DRGs.

Using the method described in Theorem 3.1 we obtained all DRGs for which the rank of the permutation representation of the group is at most 15.

Theorem 3.7 *Up to isomorphism there are exactly seven strongly regular graphs and exactly one distance-regular graphs of diameter* $d \geq 3$ *admitting an transitive action of the group* U(5, 2)*, having the rank at most 15. The constructed SRGs have parameters* $(165, 36, 3, 9)$*,* $(176, 40, 12, 8)$*,* $(297, 40, 7, 5)$*,* $(495, 238, 109, 119)$*,*
$(528, 255, 126, 120)$*,* $(1408, 567, 246, 216)$ *and* $(1782, 416, 100, 96)$*, and the DRG of diameter* $d \geq 3$ *have* 891 *vertices. Details about the obtained strongly regular graphs are given in Table 7 and details about the obtained DRG with* $d \geq 3$ *are given in Table 8.*

Graph Γ	Parameters	$Aut(\Gamma)$
$\Gamma_1^3 = \Gamma(\mathrm{U}(5,2), H_1^3)$	(165,36,3,9)	$\mathrm{U}(5,2) : Z_2$
$\Gamma_2^3 = \Gamma(\mathrm{U}(5,2), H_2^3)$	(176,40,12,8)	$\mathrm{U}(5,2) : Z_2$
$\Gamma_3^3 = \Gamma(\mathrm{U}(5,2), H_3^3)$	(297,40,7,5)	$\mathrm{U}(5,2) : Z_2$
$\Gamma_4^3 = \Gamma(\mathrm{U}(5,2), H_4^3)$	(495,238,109,119)	$O^-(10,2) : Z_2$
$\Gamma_5^3 = \Gamma(\mathrm{U}(5,2), H_5^3)$	(528,255,126,120)	$O^-(10,2) : Z_2$
$\Gamma_6^3 = \Gamma(\mathrm{U}(5,2), H_7^3)$	(1408,567,246,216)	$\mathrm{U}(6,2) : Z_2$
$\Gamma_7^3 = \Gamma(\mathrm{U}(5,2), H_8^3)$	(1782,416,100,96)	$Suz : Z_2$

Table 7: SRGs constructed from the group $\mathrm{U}(5,2)$.

Graph Γ	Vertices	Diameter	Intersection array	$Aut(\Gamma)$
$\Gamma_8^3 = \Gamma(\mathrm{U}(5,2), H_6^3)$	891	3	$\{42, 40, 32; 1, 5, 21\}$	$\mathrm{U}(6,2) \times S_3$

Table 8: DRG constructed from the group $\mathrm{U}(5,2)$, $d \geq 3$.

Remark 3.8 *The strongly regular graphs Γ_1^3, Γ_2^3, Γ_3^3 and Γ_6^3 are isomorphic to the graphs constructed in [8]. The graphs Γ_1^3, Γ_2^3, Γ_3^3, Γ_4^3, Γ_5^3 and Γ_7^3 are rank 3 graphs. The graph Γ_4^3 is known as $O^-(10,2)$ graph, i.e. the graph that belongs to the family of graphs constructed in the following way. Take the elliptic points on a nondegenerate quadric in $PG(5,2)$, where the points are adjacent when orthogonal. The graph Γ_5^3 belongs to the family of graph that can be constructed by taking the nonisotropic points of $O^-(10,2)$, adjacent when joined by a tangent. Both graphs, Γ_4^3 and Γ_5^3 are described in [15]. The SRG Γ_7^3 is known as a locally $G(2,4)$ graph. See [16] for more information.*

Remark 3.9 *The distance-regular graph Γ_8^3 have 891 vertices and diameter* 3 *is known as dual polar graph ${}^2A_6(2)$. It is the unique DRG with this intersection array. See [4] for more information.*

Acknowledgement
This work has been fully supported by Croatian Science Foundation under the project 6732. The author would like to thank Dean Crnković for his helpful comments.

References

[1] R.C. Bose and T. Shimamoto, Classification and analysis of partially balanced incomplete block designs with two associate classes, *J. Amer. Statist. Assoc.* **47** (1952), 151–184.

[2] W. Bosma and J. Cannon, *Handbook of Magma Functions*, Department of Mathematics, University of Sydney, 1994. http://magma.maths.usyd.edu.au/magma.

[3] A.E. Brouwer, *Strongly Regular Graphs*, Chapter VII.11 in "Handbook of Combinatorial Designs 2nd ed.", C.J. Colbourn and J.H. Dinitz, Eds., Chapman & Hall/CRC, Boca Raton, 2007.

[4] A.E. Brouwer, A.M. Cohen and A. Neumaier, Distance-Regular Graphs, Springer-Verlag, Berlin, 1989.

[5] P.J. Cameron, *Coherent configurations, association schemes and permutation groups*, in "Groups, combinatorics & geometry" (Durham, 2001), 55–71, World Sci. Publ., River Edge, NJ, 2003.

[6] J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker and R.A. Wilson, *Atlas of Finite Groups*, Oxford University Press, Eynsham, 1985.

[7] D. Crnković and V. Mikulić, Block designs and strongly regular graphs constructed from the group $\mathrm{U}(3,4)$, *Glas. Mat. Ser. III* , **41 (61)**, No. 2 (2006), 189–194.

[8] D. Crnković and V. Mikulić Crnković, On some combinatorial structures constructed from the groups $L(3,5)$, $\mathrm{U}(5,2)$ and $S(6,2)$, *Int. J. Comb.*, 2011 (2011), Article ID 137356, 12 pages.

[9] D. Crnković, V. Mikulić and B.G. Rodrigues, Some strongly regular graphs and self-orthogonal codes from the unitary group $\mathrm{U}(4,3)$, *Glas. Mat. Ser. III*, **45(61)** (2010), 307–323.

[10] D. Crnković, V. Mikulić Crnković and A. Švob, On some transitive combinatorial structures constructed from the unitary group $\mathrm{U}(3,3)$, *J. Statist. Plann. Inference* **144** (2014), 19–40.

[11] D. Crnković, S. Rukavina and A. Švob, New strongly regular graphs from orthogonal groups $O^+(6,2)$ and $O^-(6,2)$, *Discrete Math.*, **341** (2018), 2723–2728.

[12] D. Crnković, S. Rukavina and A. Švob, On some distance-regular graphs with many vertices, J. Algebr. Comb. (2019) https://doi.org/10.1007/s10801-019-00888-5.

[13] E.R. van Dam, J.H. Koolen and H. Tanaka, Distance-Regular Graphs, *Electron. J. Combin.* (2016), DS22, Dynamic Survey, 156 pp.

[14] *The GAP Group, GAP – Groups, Algorithms, and Programming*, Version 4.8.7; 2017. http://www.gap-system.org

[15] J.H. van Lint and A.E. Brouwer, *Strongly regular graphs and partial geometries*, in "Enumeration and Design", D.H. Jackson, A. Vanstone, Eds., Academic Press, London, 1984, pp. 85–122.

[16] D. Pasechnik, Geometric characterization of graphs from the Suzuki chain, *Europ. J. Combin.* **14** (1993) 491-499.

[17] D. Robinson, *A Course in the Theory of groups*, Springer-Verlag, New York, Berlin, Heidelberg, 1996.

[18] V.D. Tonchev, *Combinatorial Configurations: Designs, Codes, Graphs*, John Willey & Sons, New York, 1988.

[19] H. Wielandt, *Finite permutation groups*, Academic Press, New York, 1964.

[20] R.A. Wilson, *The finite simple groups*, Springer-Verlag, London, 2009.